G.P Women's College

D.M University

Mathematics (Hons) 2nd Sem

CMA – 105

Real Analysis

F.M-50

Answer all questions

1. Define countable and uncountable sets. Write down the order structure of a field. (3+7M)

   OR

   Explain the field structure of a set. (10M)

2. Define limit point of a set. Prove that every infinite bounded set has a limit point. (2+8M)

   OR

   Prove that the real number field *R* is an archemedian. For all real number *x* and *y*, show that

   $|x+y| \leq |x|+|y|$. (6+4M)

3. Define the convergence of a sequence. Prove that a sequence cannot converge to more than one limit. (2+8M)

OR

Prove that every bounded sequence has a limit point. (2+8M)

4. Define absolute convergence and conditional convergence. Prove that every absolutely convergence series is convergent. (10M)

OR

Test the series : $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \ldots\ldots..$ (10M)

5. State and prove Leibnitz's test for the convergence of an alternating series. Show that the series

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is convergent. (1+5+4M)

OR

Test the series $\sum Un$, where (i) $Un = \frac{2n+1}{3n^2+1}$

(ii) $Un = \frac{\sqrt{n+1}-\sqrt{n}}{n}$ (5+5M)

2021

(June)

Dhanamanjuri University

G.P. Women's College, Imphal

Mathematics-(Hon)

Differential Equations (CMA-107)

Total Marks = 50

Note: *Questions 1 & 2 are compulsory. Answer any four of the remaining (3-7). The figures on the right margin indicate full marks for the questions:*

1. Choose and rewrite the correct answer for each of the following: $(1\times 5=5)$

(a) The order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^{3/2}+5\left(\dfrac{dy}{dx}\right)^4+8y=\log x$ is:

(i) (3, 4) (ii) (2, 3) (iii) (4, 3) (iv) (3, 1)

(b) The spherical complementary function of the differential equation $y''+2y'+y=0$ is:

(i) $(c_1+c_2x)e^{-x}$, (ii) $(c_1x+c_2x^2)e^{-x}$, (iii) $(c_1+c_2x)e^{x}$, (iv) $(c_1x+c_2x^2)e^{x}$

(c) The integrating factor of the group of terms as being part of an exact differential equation $xdy-ydx$ is:

(i) x^2, (ii) y^2, (iii) $\dfrac{1}{y^2}$, (iv) $\dfrac{1}{x^2}$

(d) In the Lake Pollution Model of equation $\dfrac{dM}{dt}=C_{in}(t)\otimes Q_{in}(t)-C_{out}(t)\otimes Q_{out}(t)$, the volumetric flow rate through the lake is denoted by:

(i) C, (ii) M, (iii) t, (iv) Q

(e) The EMF to a circuit is governed by the differential equation $L\dfrac{di}{dt}+Ri=E$, then its solution is given by:

(ii) $ie^{Rt/L}=\int \dfrac{L}{E}e^{Rt/L}dt+c$

(iii) $ie^{Rt/L}=\int \dfrac{E}{L}e^{Rt/L}dt+c$

(iv) $ie^{Rt/}=\int \dfrac{i}{L}e^{Rt/L}dt+c$

(v) None of the above.

2. (a) What is the Mathematical Modelling? Write the applications of Mathematical Modelling.

(b) Why Mathematical Modelling applicable to differential equations? (3+2)

3. (a) What is an exact differential equation? Show that the necessary and sufficient condition that the equation $Mx+Ny=0$ will be exact for a differential equation of first order and first degree is that $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

   (b) Solve: $(x^4y^2-y)dx+(x^2y^4-x)dy=0$. (1+5+4)

4. (a) What do you mean by SIR model for spread of disease? Explain carefully how each component of the susceptible differential equations in the case of corona virus epidemic.

   (b) Find a step size for which the Euler solutions appear to closely track true solutions of the Predator-prey model system. (5+5)

5. (a) Define Clairaut's equation and find the solution of Clairaut's equation.

   (b) Show that the equation $xp^2+px-py+1-y=0$ is a Clairaut's equation. Hence obtain the general and singular solution. (4+6=10)

6. (a) Using the method of variation of parameters, solve: $(D^2+a^2)y=\sec ax$.

   (b) Solve: $(D^2-2D+2)y=e^2\tan x$ using the method of variation of parameters.

   (5+5)

7. (a) Find the Particular Integration of the form $\frac{\sin ax}{f(D)},\frac{\cos ax}{f(D)}$ and model the equation.

   (b) Solve: $(D^2-4D+13)y=\cos 2x$. (5+5=10)

2021

(June)

Dhanamanjuri University

G.P. Women's College, Imphal

Mathematics-(Pass)

Differential Equations (DMA-103)

Total Marks = 50

Note: *Answer any five of the following:*

1. Define homogeneous linear equations or Cauchy-Euler equations.

   (a) Solve: $(x^2D^2 - xD + 2)y = x\log x$

   (b) $x^3(\frac{d^3y}{dx^3}) + 2x^2(\frac{d^2y}{dx^2}) + 2y = 10(x + \frac{1}{x})$ (1+4+5)

2. (a) What is an integrating factor? Show that if the given equation $Mdx + Ndy = 0$ is homogeneous and $Mx + Ny \neq 0$, then $\frac{1}{(Mx+Ny)}$ is an integrating factor.

   (b) Solve: $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$. (1+4+5)

3. (a) Define Clairaut's equation and find the solution of Clairaut's equation.

   (b) Reduce the equation $(px - y)(x - py) = 2p$ to Clairaut's form by substitution $x^2 = u$ and $y^2 = v$ and find its complete primitive (general solution) and its singular solution, if any. (5+5=10)

4. (a) Show that the Wronskian of two solutions of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0, a_0(x) \neq 0, x \in (a,b)$ is either identically zero or never zero on (*a, b*).

   (b) Show that e^{2x} and e^{3x} are linearly independent solutions of $y'' - 5y' + 6y = 0$. Find the solution $y(x)$ with the property that $y(0) = 0$ and $y'(0) = 1$. (5+5=10)

5. (a) Write the general (standard) form of linear equations of the second order. Prove that $y = \sin x$ is a part of the complementary function (c.f.) of the equation $(\sin x - x\cos x)y'' - x\sin x y' + y\sin x = 0$.

   (b) Solve: $x^2y'' - (x^2 + 2x)y' + (x+2)y = x^3e^x$. (5+5=10)

6. (a) Reduce the differential equation $y'' + Py' + Qy = R$, where *P*, *Q* and *R* are functions of x to the form $\frac{d^2v}{dx^2} + Iv = S$, which is known as the normal form of the given equation.

   (b) Apply the method of variation parameters to solve $y_2 + y = \operatorname{cosec} x$. (5+5=10)

2021

(June)

Dhanamanjuri University

G.P. Women's College, Imphal

Mathematics-(Generic)

Differential Equations (GEM-003)

Total Marks = 50

Note: *Answer any five of the following:*

1. (a) What is an exact differential equation? To determine the necessary and sufficient condition for a differential equation of first order and first degree to be exact.

   (b) Solve: $\{y(1+\frac{1}{x})+\cos y\}dx+(x+\log x-x\sin y)dy=0$. (1+5+4)

2. (a) Define an integrating factor. Prove that if $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$ is a function of x alone say $f(x)$, then $e^{\int f(x)dx}$ is an integrating factor of $Mdx+Ndy=0$.

   (b) Using the above rule, solve $(x^2+y^2+x)dx+xydy=0$. (1+5+4)

3. (a) Define Clairaut's equation and find the solution of Clairaut's equation.

   (b) Reduce the equation $x^2p^2+py(2x+y)+y^2=0$ where $p=\frac{dy}{dx}$ to Clairaut's form by putting $u=y$ and $v=xy$ and find its complete primitive and its singular solution. (4+6=10)

4. (a) Define the Wronskian. Prove that two solutions $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y''+a_1(x)y'+a_2(x)y=0, a_0(x)\neq 0, x\in(a,b)$ are linearly dependent iff their Wronskian is identically zero.

   (b) Prove that $\sin 2x$ and $\cos 2x$ are solutions of the differential equation $y''+4y=0$ .and these solutions are linearly independent.

   (1+5+4)

5. (a) Find the complete solution of $y''+Py'+Qy=R$ in terms of one known integral belonging to the complementary function (c.f.).

   (b) Solve: $xy''-(2x-1)y'+(x-1)y=0$. (6+4=10)

6. (a) By using transformation of the equation to the normal (i.e. removal of the first derivative) solve $y''-2\tan xy'+5y=\sec xe^x$.

   (b) Apply the method of variation parameters to solve $y_2+n^2y=\sec nx$. (5+5=10)