2021
(JUNE)

MATHEMATICS
HONOURS

MAT-308

(Real Analysis)

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

1. Answer (a) or (b) **10x1=10**

a) i) State and prove Archimedean propertry. 1=3=4

ii) State and prove Heine-Borel theorem. 1+5=6

b) i) State and prove Bolzano-Weierstrass theorem(for set). 1+5=6

ii) Prove that the union of an arbitrary family of open sets is open. Is arbitrary intersection of open sets necessarily open? Give an example in support of your answer. 4

2. Answer (a) or (b). **10x1=10**

a) i) Prove that the sequence $\{u_n\}$ converges to its supremum if it is bounded and $u_{n+1} \geq u_n \forall n$. Hence or otherwise, show that the sequence $\{u_n\}$ where $u_n = \frac{3}{n+n} + \frac{3}{n+(n-1)} + \cdots + \frac{3}{n+2} + \frac{3}{n+1}$ is convergent. 5

ii) State and prove nested interval theorem. 5

b) i) If f is continuous function in $[a, b]$and $f(a)f(b) < 0$, then there exists a point $c \in]a, b[$ such that $f(c) = 0$. Prove the same. 5

ii) Show that if a function f is continuous in $[a, b]$, then it is uniformly continuous in $[a, b]$. 5

3. Answer (a) or (b). **10x1=10**

a) Let $|f(x)| \leq K$ for all x in $[a, b]$ and P be a partition of $[a, b]$ with norm$\leq \delta$. If P^* is a refinement of P containing at most q more points than P, prove that
$\cup (P^*, f) \leq \cup (P, f) \leq \cup (P^*, f) + 2kq\delta$.
Hence prove Darbour's theorem on upper R-integral. 10

b) i) Prove that the oscillation of a bounded function f on an interval $[a, b]$ is the supremum of the set $\{|f(x_1) - f(x_2)|: x_1, x_2 \in [a, b]\}$. 5

ii) Prove that a bounded function f which has only a finite number of points of discontinuity in a closed interval $[a, b]$ is intergrable in $[a, b]$. 5

4. Answer (a) or (b). **10x1=10**

a) i) Test the convergence of $\int_0^1 \log x^7 dx$. 3

ii) Examine the convergence of the improper integral $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ and hence determine the value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$. 7

b) State and prove Frullani's improper integral theorem. By using the same, show that $\int_0^\infty \frac{tan^{-1}ax - tan^{-1}bx}{x}\, dx = \frac{\pi}{2}\log\frac{b}{a}$ 1+6+3=10

5. Answer (a) or (b) **10x1=10**

a) i) Show that the function f, where

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & \text{if } x^2+y^2 \neq 0 \\ 0 & \text{if } x = y = 0 \end{cases}$$

possesses partial derivative but not differentiable at the origin. 4

ii) State and prove Young's Theorem on the reversal of the order of the partial derivation. 1+5=6

b) i) Show that the function f, where

$$f(x,y) = \begin{cases} \frac{x^3y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

possesses partial derivative but is not differentiable at the origin. 4

ii) State and prove Schwarz's theorem on the reversal of the order of partial derivatives. 1+5=6

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2021
(JUNE)

MATHEMATICS
HONOURS

MAT-309

(Metric Space, Calculus of Variation & Rigid Dynamics)

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

SECTION-A

Answers any five questions: **5 X 5 = 25**

1. Define an open set and prove that a subset G of a metric space X is open if and only if it is a union of open spheres.
2. Let X be a metric space. Then prove that
   a) any union of open sets in X is open
   b) any finite intersection of open sets in X is open.
3. Show that the trivial metric on a set X is a metric i.e. the distinct function $d: X * X \to R$ defined by
   $$d(a,b) = \begin{cases} 1 & if\ a \neq b \\ 0 & if\ a = b \end{cases}$$
   is a metric.
4. Prove that a subset F of a metric space X is closed if and only its compliment F' is open.
5. State and prove Holder's Inequality.
6. Define a complete metric space. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.
7. State and prove Cantor's Intersection Theorem.
8. Prove that a mapping f from a metric space X into a metric space Y is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.
9. Define the following:
   i) Homeomorphism
   ii) Nowhere dense
   iii) Boundary Point
   iv) Dense Subset
   v) Neighbourhood of a point.
10. Show that a Cauchy sequence is convergent if it has a convergent subsequence.

SECTION-B

Answer any five questions: **5 X 5 = 25**

11. State and prove the Branchistochrone problem in Calculus of Variation.

12. Find the extremals of the functional

$$\int_0^{\pi/2} \left\{2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} dt$$

with conditions $x(0) = 0,$ $x\left(\pi/2\right) = -1$

$y(0) = 0,$ $y\left(\pi/2\right) = 1$

13. State and prove the theorem of parallel axes on a rigid body.

14. State D' Alembert's Principle and discuss the general equation of motion of a rigid body using this principle.

15. If a rigid body swings under gravity from a fixed horizontal axis, show that the time of complete oscillation is

$$2\pi\sqrt{\frac{\mathrm{R}^2}{h_g}},$$ where Ʀ is its radius

of gyration about the fixed axis and h is the distance between the fixed axis and the centre of inertia of the body.

16. A solid homogeneous cone of height h and vertical angle $2 \propto$ oscilates about a horizontal axis through its vertex. Show that the length of simple pendulum is $\frac{1}{5}h(4 + tan^2 \propto)$.

17. A rough uniform board of mass m and length $2a$ rest on a smooth horizontal plane and a man of mass M walks on it from one end to other. Find the distance through which the board moves in this time.

18. A uniform sphere rolls down an inclined plane rough enough to prevent any sliding, discuss the motion.

19. Show that the momental ellipsoid at the centre of an ellipsoid is $(b^2 + c^2)x^2 + (a^2 + c^2)y^2 + (a^2 + b^2)z^2 = constant.$

20. Find the equation of motion in two dimension when the forces acting on the body are finite.

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2021
(JUNE)

MATHEMATICS
HONOURS

MAT-310

Special Theory of Relativity & Tensor

Full Marks: 50
The figures in the margin indicates full marks for the questions
Answer all the questions.

SECTION-A

Question Nos. 1 & 2 are compulsory and choose any 3 (three) questions from the remaining of Section-A

1. a) The basic theory of field is governed by: 1x5=5
   - i) Laplace Transformation
   - ii) Lorentz Transformation
   - iii) Legendre's Transformation
   - iv) Lagrange's Transformation

   b) Lorentz Transformation reduces to Galilean one if:
   - i) $v = c$
   - ii) $v \ll c$
   - iii) $v \gg c$
   - iv) None of the above.

   c) Aberration of light from stars is caused due to:
   - i) the travelling of light in the atmosphere
   - ii) the scattering of light by the air particles
   - iii) elliptical orbit of the earth around the sun
   - iv) None of the above.

   d) The relation between momentum and energy is:
   - i) $E^2 = p^2c^2 - m_0^{\ 2}c^4$
   - ii) $E^2 = p^2c^2 - m_0^{\ 2}c^2$
   - iii) $E^2 = p^2c^2 + m_0^{\ 2}c^4$
   - iv) $E^2 = p^2c^2 + m_0^{\ 2}c^2$

   e) The resultant of two velocities of light each of which is less than c is also:
   - i) greater than c
   - ii) equal to c
   - iii) $v^2 > c^2$
   - iv) less than c.

2. a) What do you mean by binding energy? 2

   b) Calculate the binding energy of one helium nucleus, if the mass of a hydrogen atom is 1.00814 amu and that of a helium atom is 4.00388 amu. 4

3. a) Prove that $m = \frac{m_0}{\left(1 - \frac{u^2}{v^2}\right)^{1/2}}$, where the symbols have their usual notations. 5

   b) The length of a rocket ship is 100 metres on the ground. When it is in flight its length observed on the ground is 99 metres. Find the speed of the rocket. 3

4. a) Show that the apparent length is contracted by the factor $\sqrt{\left(1 - \frac{u^2}{v^2}\right)}$ in the direction of relative motion $\left(l' < l\right)$ 5

   b) Show that is $x^2 + y^2 + z^2 - c^2t^2$ Lorentz invariant. 3

5. a) Prove that $\left(p^2 - \frac{E^2}{c^2}\right)$ is Lorentz invariant. 5

   b) Suppose the half-life of a certain particle is 10^{-7} second, when it is at rest. What will be its half-life when it is travelling with a speed of 0.99*c*? 3

6. a) Deduce the transformation equations for momentum? 5

   b) The rest mass of an electron is 9×10^{-7} gm. What will be mass if it were moving with velocity $\frac{4}{5}$ times the speed of the light? 3

SECTION-B

Questions No. 7 is compulsory and any 2 (two) questions from the remaining Section-B

7. a) What is the Contravariant tensor of rank one or first order?

   b) Define the Kronecker delta.

   c) Write the term of the following indicated sums $\frac{\delta}{\delta x^k}\left(\sqrt{(g)}A^k\right), N = 3$.

   d) Write the law of transformation for the tensor B^k_{jk}.

   e) What is symmetric tensor? 1x5=5

8. The outer product of two tensors in a tensor whose rank (or order) is the sum of the ranks of the two tensors. 5

9. A symmetry tensor of rank two has at most $\frac{1}{2}N\left(N+1\right)$ different components in V_N. 5

10. a) If A_{ij} is a skew-symmetric tensor. Prove that $\left(\delta^i_j\delta^k_l + \delta^i_l\delta^k_j\right)A_{ik} = 0$

    b) Suppose g_{ij} and g^{ij} are reciprocal symmetric tensors of the second order.

    Then prove that $g^{ij}\frac{\delta g_{ij}}{\delta x^k} + g_{ij}\frac{\delta g^{ij}}{\delta x^k} = 0$. 3+2=5

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2021
(JUNE)

MATHEMATICS
HONOURS

MAT-311

Mathematical Statistics (Optional)

Theory

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

1. Obtain a formula for median of Exponential distribution with parameter λ>0. Show that moment generating function for a exponent variate with parameter λ>0 is
$$1+\frac{t}{\lambda}+\left(\frac{t}{\lambda}\right)^2+\cdots$$ 10

Or

On x-axis (n+1) points are taken independently between the origin and x=1, all positions being equally likely. Show that probability that the $(k+1)^{th}$ of these points, counted from the origin, lies in the interval $x-\frac{1}{2}dx$ to $x+\frac{1}{2}dx$ is $\binom{n}{k}(n+1)x^k(1-x)^{n-k}dx$. 10

2. If $E(X)=\sum xP_x$ exists prove that $E(X^2)=P''(1)+P'(1)=2Q'(1)+Q(1)$ and $V(X)=2Q'(1)+Q(1)-\{Q(1)\}^2=P''(1)+P'(1)-\{P'(1)\}^2$. 10

Or

"A random variable X may have no moments although its moment generating function exists". Justify the above statement with an example. 10

3. Define convergence of random variable in probability. Let $\{X_n\}$ be a sequence of random variable such that $X_n\to X$ and $X_n\to X'$. Prove that $X=X'$. 10

Or

Let $\{X_n\}$ be a sequence of random variable such that $X_n\xrightarrow{r}X$ Prove that $E|X_n|\xrightarrow{r}E|X|$. 10

4. Prove that mean=mode= median for normal distribution. 10

Or

Draw the normal density curve and give six geometrical significances of this curve. 10

5. State and prove Chebyshev's inequality. 10

Or

State and prove De-Moivre-Laplace central limit theorem. 10

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2021
(JUNE)

MATHEMATICS
HONOURS

MAT-312

(Optional-Spherical Trigonometry and Astronomy)

Theory

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer all the questions.

1. Choose and rewrite the correct answer for each of the following: **1x5=5**

a) In a spherical triangle *ABC*, the sum *A*+*B*+*C* is

i) greater than 0 and less than π

ii) greater than π and less than 2π

iii) greater than 2π and less than 3π

iv) equal to π

b) The spherical excess (*E*) of a spherical triangle *ABC* is

i) $A+B+C+\pi$

ii) $A+B+C+\frac{\pi}{2}$

iii) $A+B+C-\pi$

iv) $A+B+C-\frac{\pi}{2}$

c) The sun is at the first point of Libra on the

i) Vernal equinox

ii) Autumnal solstice

iii) Winter solstice

iv) Summer solstice

d) The right ascension of the sun on the 23rd September is

i) 0^o

ii) $23^o27'$

iii) 90^o

iv) 180^o

e) The area of a spherical quadrilateral ABCD is

i) $(A+B+C+D-\pi)r$

ii) $\left(A+B+C+D-\frac{\pi}{2}\right)r^2$

iii) $(A+B+C+D-2\pi)r^2$

iv) $(A+B+C+D-2\pi)r^2$

2. Write very short answer for each of the following: 1x6=6

a) Write the sine formula in a spherical triangle.

b) Name the two kinds of parallax

c) Distinguish between annual aberration and diurnal aberration.

d) What is the spherical excess of a spherical triangle *ABC*?

e) What is Earth's way?

f) What is precession?

3. Write short answer for any *three* of the following: 3x3=9

a) In any equilateral spherical triangle *ABC*, show that

$$secA = 1 + \sec\alpha$$

b) State Kepler's three laws of planetary motion.

c) In a spherical triangle *ABC* in which angle *C* is a right angle, prove that

$$\sin(c+a)\sin(c-a) = sin^2b\, cos^2\, a = cos^2\, A\, sin^2c,$$

d) Show that the sum of the three sides of a spherical triangle is less than the circumference of a great circle.

e) Deduce Kepler's third law from Newton's law of gravitation.

4. Answer any two questions: 6x2=12

a) In any spherical triangle *ABC*, Show that

$$\frac{\sin(a+b)}{\sin c} = \frac{\cos A + \cos B}{1 - \cos C}$$

b) State and prove a cotangent formula in a spherical triangle.

c) In a spherical triangle *ABC*, prove that

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

5. Answer any one question: 6x1=6

a) If two stars (α, δ) and (α_1, δ_1) rise at the same moment at a place in latitude ϕ, show that

$$cot^2\phi sin^2(\alpha_1 - \alpha) = tan^2\delta + tan^2\delta_1 - 2\tan\delta\, tan\delta_1 \cos(\alpha_1 - \alpha).$$

b) Two stars (α_1, δ_1) and (α_2, δ_2) have the same longitude; prove that

$$sin(\alpha_1 - \alpha_2) = tan\varepsilon(\cos\alpha_1 \tan\delta_2 - \cos\alpha_2 \tan\delta_1).$$

6. Answer any one question. 6x1=6

a) Find the coefficient of refraction by Bradley's method.

b) What do precession and nutation have in common? Explain the physical cause of nutation.

7. Answer any one question. 6x1=6

a) Show that the apparent path of a star on account of aberration is an ellipse.

b) Discuss the right ascension and declination due to geocentric parallax where earth is taken as spheroid.

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2021
(JUNE)

MATHEMATICS
HONOURS

MAT-314

(Computational Mathematics Laboratory)
Theory

Full Marks: 50

The figures in the margin indicates full marks for the questions
Answer any ten questions.

1. Define variable. Explain any four system variables. 1+4=5

2. Define the following terms: 1x5=5

   i) workspace ii) who
   iii) what iv) which
   v) clear

3. Create a 3x3 matrix by concatenating 3 arrays. And write the command to access 3+2=5

   i) elements of 2^{nd} row ii) elements of 3^{rd} column

4. Generate a 4x4 matrix. Do the following: 1x5=5

   i) write the command to insert 5^{th} row
   ii) write the command to delete the 3^{rd} column
   iii) write the command to flip left- right direction
   iv) write the command to filp up-down direction
   v) write the command to rotate the matrix 90 degree

5. Find the lower and and upper triangular matrix of a magic square matrix 2½ x 2=5

6. Define script file. Write a script file to find the difference of two variable and display the result. 1+4=5

7. Write a script file to find the compound interest. Display the answer using fprintf function. 5

8. Write a script file to plot the graph of the function y=3x+7. Label the x-axis, y-axis and give title. 5

9. Write the MATLAB command to solve the following system of linear equations by matrix operation: 5

   $4x-2y+7z=5$
   $2x+8y+2z=10$
   $5x+6y+4z=8$

10. Let A= [1 5 6;2 3 4;11 7 9; 3 4 5]. Write the output of the following command 5

    i) `>> A(2,1)` ii) `>> A(3,: )`
    iii) `>> A(2: 4,1: 2)` iv) `>> A(1: 2,: )`
    v) `>> A(: , 1)`

11. Explain the uses of beak and continue with examples. 5

12. Write a script file using for loop to print first n natural numbers. Take the value of n from user. 5

13. Define function file in MATLAB. Create a function file to display "WELCOME TO MATLAB". 1+4=5

14. Write a function file to compute mean and median of an input array. 5

15. Write the command to generate 3D plot for the given functions. Also label the axes and give a title. $x=t^2$ and $y=4t$ where t= -3 to 10 with an interval 0.2. 5

16. Suppose A=[12 23 15 10;7 9 10 3;2 4 6 8; 10 7 5 6]. Append the average of each row and each columns as two extra rows. 5

17. Write a script file to display the result according to the marks score by a student. Take the mark from the user. 5

18. Write a script file to find the area of a circle and area of a triangle. 5

19. Plot the following two functions in a graph using subplot function. 5

$$\mathrm{y} = e^{-1.5}x \sin(10x) and\ y = e^{-2}\sin(10x)$$

20. Write a script using while loop to find the sum of first n positive numbers which are divisible by 5. 5

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